

LOCKHEED MISSILES & SPACE COMPANY
HUNTSVILLE RESEARCH & ENGINEERING CENTER
HUNTSVILLE RESEARCH PARK
4800 BRADFORD DRIVE, HUNTSVILLE, ALABAMA

A METHOD FOR CALCULATING
FLOW FIELD PROPERTIES
IN LOW DENSITY PLUMES

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by

S. J. Robertson

APPROVED BY: John W. Benefield
J. W. Benefield, Supervisor
Aero-Physics Section

B. D. Stator / for
D. C. Shea, Manager
Aero-Mechanics Department

FOREWORD

This document presents the results of work performed in the Aero-Physics Section, Aero-Mechanics Department, of Lockheed's Huntsville Research & Engineering Center in support of Marshall Space Flight Center Contract NAS8-21150. The NASA Technical Monitor for this contract is W.O. Randolph, R-P&VE-PT.

SUMMARY

An investigation was made to develop a technique for calculating the flowfield properties of jet plumes exhausting into a near vacuum environment. This technique is intended to be used to obtain heating and pressure loadings on vehicle surfaces at high altitudes. From a survey made of previous investigations and existing techniques, an approach was selected which is most likely to yield the most immediate, yet reasonably accurate, results. The selected method uses an existing method-of-characteristics computer program modified to include a "sudden freeze" criterion for treating the non-equilibrium condition of the various energy modes (vibration, rotation, translation). This criterion specifies a point along a streamline at which the temperature associated with a particular energy mode freezes and remains constant. When the translational "freeze point" is reached, the flow is considered to be free molecular from that point on, with velocity and temperature remaining constant and the streamlines straight. The density then is made to vary as the inverse of the streamtube cross sectional area.

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Section 1

INTRODUCTION

The use of attitude control and retro rockets at high altitudes and in space has resulted in increased interest in flowfield properties of jet plumes from highly underexpanded nozzles in a near vacuum environment. Large billowing plumes, in some cases with the flow direction turning upstream around the nozzle exit, are characteristic of rocket exhausts in these environmental conditions. Of interest to vehicle design are the heating and pressure loading and material deposition resulting from impingement of these billowing plumes on nearby vehicle surfaces.

As the exhaust gases expand in the plume, the pressure and density decrease, and, as a result, the intermolecular collision frequency decreases and the mean free path between intermolecular collisions increases. Eventually, the intermolecular collisions become insufficient to effect changes in the translational temperature as the gas flows downstream, and, therefore, the gas may no longer be treated as a continuous fluid. In order to calculate the flow properties throughout the plume, reliable criteria must be used for defining the extent of the continuum flow region. Special techniques must then be used for treating the remaining noncontinuum region.

This report presents the results of a study of previous investigations and existing techniques for calculating flowfield characteristics in low density rocket exhaust plumes. A technique is presented which is considered appropriate for most engineering applications.

Section 2

LITERATURE SURVEY

Most previous studies of jets exhausting into vacuums have been directed toward the development of supersonic molecular beams. The first studies of this nature were by Owen and Thornhill (Reference 1) and Sherman (Reference 2) who used continuum flow techniques (method of characteristics) to calculate the flow field out to centerline Mach numbers as high as 100. Owen and Thornhill developed an empirical relationship between the centerline Mach number and the distance along the centerline which has been much used in molecular beam studies. Sherman noted in his investigation that a free jet would become inertia-dominated at some distance from the nozzle in such a manner that the streamlines would become straight and strongly resemble point-source flow.

Vick, et al., (Reference 3) used the method of characteristics to calculate flowfield characteristics of jet exhaust plumes from highly under-expanded nozzles for exit to ambient pressure ratios up to 10^7 . These calculated jet boundaries were found to be in good agreement with experimentally observed boundaries in the near flow field. The experimentally observed boundaries were taken from schlieren photographs.

Cassanova and Stephenson (Reference 4) made an investigation similar to that of Vick, et al., in which calculated method-of-characteristics flow-field properties were compared with experimentally observed values. In this study, streamline directions were experimentally observed using vanes, and densities were observed from pitot tube measurements in the continuum region and from normal flat plate drag force measurements in the noncontinuum region. As with Vick, et al., good agreement was noted in all cases in the near flow (continuum) region. Also noted was that, if transition to noncontinuum flow occurred in the source-like flow region (Mach numbers high and streamlines nearly straight), good agreement was obtained into the noncontinuum

region. This phenomenon is explained as being caused by the high Mach numbers in the source-like flow region. At high Mach numbers, the Mach angle is quite small, and, hence, there is very little lateral interaction in the flow field. Also, the velocity has nearly reached its terminal value; thus, the density varies as the inverse of a streamtube cross sectional area. In this region, therefore, the flow field which is calculated using continuum techniques closely resembles free molecular flow.

A comprehensive comparison study was also carried out by Lockheed/Huntsville (Reference 5) to verify the use of the method of characteristics over a wide range of pressure ratios. This study, correlating Schlieren photographs of plume boundaries, also indicates excellent agreement with observed properties in the near plume field.

Marrone (Reference 6) made an experimental investigation of low density nitrogen jets exhausting into a near vacuum. Rotational temperature and density measurements were made using the electron beam fluorescence technique. Throughout the range of measurements, density measurements were found to agree with theoretical isentropic flow values. The rotational temperature, however, was found to depart from the theoretical values at low temperatures and to freeze at a constant value. This phenomenon is due to the decreasing number of intermolecular collisions as the gas expands to low densities. According to the equipartition of energy, a gas in thermal equilibrium maintains the same temperature in each of the available energy modes in the gas. For nitrogen at the temperatures encountered in this study, the energy modes are translation and rotation. If the gas undergoes a sudden change in the translational temperature, approximately five to ten collisions per molecule are required to bring the rotational temperature back into equilibrium with the translational temperature. During a rapid expansion at low densities, therefore, the translational temperature may decrease at a much faster rate than the rotational temperature can follow, and the rotational temperature may be considered frozen. This argument is also valid for gases having vibrational energy modes. In fact, the vibrational modes will freeze much sooner than the rotational modes, since from approximately 1000 to several million collisions per molecule are required to equilibrate vibration with translation.

In analogy with the preceding discussion of vibrational and rotational freezing, the translational temperature may also be considered to freeze when the intermolecular collisions are no longer sufficient to effect changes in translational temperature. This, in effect, means that the flow has become free molecular.

Although there is in reality a continuous transition from equilibrium to frozen flow for each of the temperatures, a "sudden freeze" approximation may be made with reasonable accuracy. Knuth (References 7 and 8) defined criteria for both rotational and translational freezing in jets based on the following equation:

$$\left| \frac{DT}{Dt} \right| = \frac{T}{\tau} \quad (1)$$

where D/Dt is the hydrodynamic derivative, T is temperature (vibrational, rotational, or translational), and τ is the relaxation time for the temperature to reach equilibrium after a step change. If the magnitude of DT/Dt is smaller than T/τ , the temperature is considered to be in equilibrium; if DT/Dt is greater than T/τ , it is considered to be frozen. This criterion is analogous to previously investigated nonequilibrium effects, such as dissociation or chemical reaction (Bray, Reference 9).

The physical meaning of Equation (1) may be seen by observing that $T/|DT/Dt|$ is an indication of the time involved in an appreciable change in temperature of the gas as it flows downstream, assuming equilibrium continuum flow. Now, τ is the relaxation time required for the gas temperature to reach equilibrium after a step change (such as passing through a shock wave), and may be considered a minimum time interval required to effect any change in the gas temperature. When τ is larger than $T/|DT/Dt|$, the gas temperature cannot relax fast enough to maintain the equilibrium rate of change, and the temperature departs from equilibrium.

For steady flow, Equation (1) may be written:

$$V \left| \frac{dT}{ds} \right| = T/\tau \quad (2)$$

where V is the flow velocity and s is distance along a streamline. The relaxation time τ may be expressed in terms of a collision number N as follows:

$$\tau = N/\dot{N} \quad (3)$$

where \dot{N} is the local collision frequency. N is simply the number of collisions per molecule required to achieve relaxation. The collision frequency \dot{N} may be expressed in terms of the mean thermal velocity \bar{V} and the Lagrangian (relative to moving gas) mean free path λ as follows:

$$\dot{N} = \bar{V}/\lambda \quad (4)$$

Rearranging Equation (2), using Equations (3) and (4), yields the freezing point criterion:

$$Kn' = \left(\frac{V}{\bar{V}} \lambda \right) \left(\frac{1}{T} \left| \frac{dT}{ds} \right| \right) = \frac{1}{N} \quad (5)$$

The dimensionless parameter Kn' on the left-hand side of Equation (5) is a local Knudsen number, which is a ratio of a mean free path to a characteristic length. The mean free path in this case is the Eulerian (relative to fixed coordinate system) mean free path $(V/\bar{V})\lambda$; and the characteristic length is $T/|dT/ds|$, the distance over which an appreciable change in temperature occurs.

Scott and Phipps (Reference 10), independently of Knuth, introduced the concept of a local Knudsen number in determining the transition from continuum flow to free molecule flow. (Actually, Knuth did not express his criterion in terms of a local Knudsen number.) They considered both temperature and

Mach number gradients in the definition of the characteristic length, however, and were concerned only with a qualitative relationship between terminal Mach number and source Knudsen number. They did not develop any quantitative criteria for freezing point locations.

Hamel and Willis (Reference 11) developed a moment solution of the Boltzman equation for spherical source flow. As was brought out in the preceding discussions, at large distances from the nozzle the flow of a free jet into a vacuum is approximated by spherical source flow. In their solution, the translational temperature was resolved into components parallel to and perpendicular to the streamline. An interesting finding in their analysis is that the perpendicular temperature initially decreases as $r^{-4/3}$ (r is distance from source), as in the isentropic case, and then abruptly changes to an r^{-1} dependency in the asymptotic limit. Since a collisionless flow should have an r^{-2} dependency, this indicates that the source flow expansion never really reaches free molecular flow. The parallel temperature, however, does approach a finite limiting value and, therefore, freezes. The terminal Mach number, based on this freezing temperature, was found to be proportional to $Kn_o^{-0.4}$ (Kn_o is source Knudsen number) for hard sphere molecules and $Kn_o^{-0.667}$ for Maxwell (inverse fifth power law repulsion) molecules. A similar analysis performed for cylindrical source flow showed that both components of temperature approach zero at infinity, and, hence, there is no temperature freeze.

Edwards and Cheng (Reference 12) independently obtained the same results as Hamel and Willis by using the B-G-K kinetic model (Reference 13).

Anderson and Fenn (Reference 14) used a more empirical approach than Hamel and Willis or Edwards and Cheng for predicting terminal Mach numbers in a free jet. Their analysis was based on the maximum decrease in Mach number allowed by a collision effectiveness parameter. This collision effectiveness parameter was introduced by Oman, et al. (Reference 15). The terminal Mach number was found by their analysis to be proportional to $Kn_o^{-0.4}$, in agreement with Hamel and Willis for hard sphere molecules.

Abuaf, et al., (Reference 16) made experimental measurements of the perpendicular temperature in free jet expansions. They noted an $r^{-4/3}$ relationship in the near flow region and an r^{-2} relationship in the asymptotic limit. These results are in contradiction to the prediction of Hamel and Willis in the asymptotic limit and cast at least some doubt on the validity of their theory. This contradiction remains to be resolved satisfactorily.

Section 3

SELECTED METHOD OF APPROACH

Based on the review of previous investigations and existing techniques, described in the preceding section, a method of approach was selected which seemed likely to yield the most immediate, yet reasonably accurate, results.

The most rigorous and potentially the most accurate approach would be to solve the Boltzman equation along the lines suggested by Hamel and Willis (Reference 11) or Edwards and Cheng (Reference 12). Their solutions, however, were for the particular case of spherical source flow and, therefore, more amenable to an elegant mathematical treatment. For a typical jet exhaust plume, a more numerical approach is required to solve the Boltzman equation. Developing this approach would be a worthwhile future project, but immediate results would not likely be forthcoming.

The method of approach selected for development in this study is to use the method of characteristics, in conjunction with the "sudden freeze" criteria of Knuth (References 7 and 8), up to the freezing point for translational temperature. From this point on, the mass flow velocity and temperature will be held constant and the streamline straight, i.e., the flow will be considered free molecular. This approach appears to be readily adaptable for use in an existing method of characteristics computer program (Reference 17) and should yield reasonably accurate results within a reasonable period of time.

3.1 LOCAL KNUDSEN NUMBER

A local Knudsen number Kn' (Equation 5) will be calculated at each point in the characteristic net:

$$\begin{aligned}
 \text{Kn}' &= \left(\frac{V}{\bar{V}} \lambda \right) \left(\frac{1}{T} \frac{dT}{ds} \right) \\
 &= \sqrt{\frac{\pi \gamma}{8}} M \frac{\lambda}{R_o} \left| \frac{d(\ln T)}{ds} \right|
 \end{aligned} \tag{6}$$

where γ is the ratio of specific heats, M is local Mach number, R_o is nozzle exit radius, and \bar{s} is distance along streamline divided by R_o .

For the special case of a nonreacting, constant collision cross-section hard sphere gas, the mean free path λ is inversely proportional to density, and Equation (6) may be expressed:

$$\text{Kn}' = \sqrt{\frac{\pi \gamma}{8}} \frac{\text{Kn}_c}{\bar{\rho}} M \left| \frac{d(\ln T)}{ds} \right| \tag{7}$$

where $\text{Kn}_c = \lambda_c/R_o$ is the chamber Knudsen number, and $\bar{\rho}$ is the ratio of local density to chamber density.

In real gases, the molecular interactions are more complex than those predicted by the simple hard sphere model. A more realistic determination of the local mean free path λ can be obtained through the viscosity μ by the following relation (Reference 18):

$$\lambda = \frac{\mu}{0.499 \rho \bar{V}} \tag{8}$$

where ρ is the local density. Equation (8) is exact only for the case of hard spheres. However, it is nearly the same for all inverse power law repulsions, with the numerical factor in the denominator being 0.491 for Maxwell (inverse fifth power law repulsion) molecules. The nature of the molecular interaction is accounted for by the variation of the effective collision cross section and, hence, the viscosity with temperature. A number of empirical formulae (Reference 18) have been devised for accounting for the variation of viscosity with temperature.

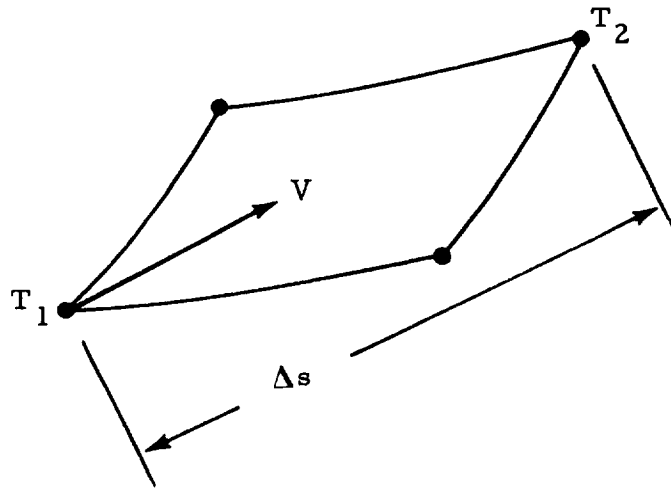
Using Equation (8), Equation (6) may now be expressed:

$$Kn' = 2.51\pi\gamma \frac{M^2}{Re} \left| \frac{d(\ln T)}{ds} \right| \quad (9)$$

where Re is the local Reynolds number:

$$Re = \frac{\rho V R_o}{\mu} \quad (10)$$

An approximation of the derivative $|d(\ln T)/ds|$ in Equation (9) is obtained by finding the difference in $\ln T$ between the diagonal points in the characteristic net:



with

$$\left| \frac{d(\ln T)}{ds} \right| \cong \frac{\ln(T_1/T_2)}{\Delta s/R_o} \quad (11)$$

The accuracy of this approximation increases with the tightness of the characteristics net.

3.2 FREEZING POINT CRITERIA

When the local Knudsen number Kn' becomes equal to the inverse of the collision number N for a particular energy mode, the temperature corresponding to that particular energy mode is considered to freeze and remain constant thereafter. The isentropic exponent γ , used in the method-of-characteristics flowfield calculations, then changes accordingly. New stagnation values of temperature, density, and pressure are then calculated such that the static values are preserved across the freezing front.

3.2.1 Vibrational Freezing

When vibrational freezing occurs, the gas takes on the value of γ corresponding to its rotational and translational degrees of freedom. For a diatomic molecule such as nitrogen or oxygen, γ becomes 7/5. For a nonsymmetrical molecule such as water vapor or carbon dioxide, γ becomes 4/3.

The vibrational collision number N_v is highly temperature-dependent and varies greatly, depending on the kind of gas under consideration. In the literature, the vibrational relaxation characteristics of a gas are usually given in terms of the relaxation time τ_v at atmospheric pressure. The collision number N_v can be obtained from the relaxation time τ_v by use of Equation (3) combined with Equations (4) and (8):

$$\begin{aligned} N_v &= 0.499 \rho_o \bar{V}^2 \tau_v \\ &= \frac{0.399}{\pi} \frac{p_o \tau_v}{\mu} \end{aligned} \tag{12}$$

where ρ_o and p_o are the density and pressure at atmospheric pressure. Reference 19 provides a large amount of data on vibrational collision numbers and relaxation times for various gases at various temperatures.

3.2.2 Rotational Freezing

After rotational freezing occurs, the gas is treated as a monatomic gas with γ equal to $5/3$. The rotational collision number N_r is only a weak function of temperature (Reference 7) and may be treated as a constant for a given gas. Rotational collision numbers are given in Reference 20 for nitrogen, oxygen, and air as 5.26, 4.09 and 4.82, respectively. Rotational collision number data is also provided in Reference 19.

3.2.3 Translational Freezing

When translational freezing occurs, the gas may be considered in free molecular flow. The streamlines remain straight thereafter, and the molecular velocities are all in the direction of stream flow. There is a distribution of velocities in the direction of flow corresponding to the frozen translational temperature, but there are no random velocities with components perpendicular to the streamline.

Knuth (Reference 7) used a translational collision number N_t of $5/4$ which was based on Maxwell's (Reference 21) calculations for pressure inequality relaxations. This is equivalent to a local Knudsen number Kn' of 0.8 (Equation 5) which is of the same order of magnitude as the Knudsen number specified for transition to free molecular flow in other rarefied gas flow studies.

3.3 FREE MOLECULAR FLOW FIELD

In the free molecular flow region, the stream velocity is constant, and the density varies according to the spreading of the straight streamlines, with the density being inversely proportional to the cross-sectional area of a stream tube.

The calculation procedure requires first finding the intersection of two adjacent extrapolated free molecular streamlines (see Figure 1). The points 1 and 2 on the figure are two adjacent points where translational freezing occurs. The intersection of the streamlines passing through these points and extrapolated into the nonfree molecular flow field is found by:

$$\tilde{X} = \frac{X_1 \tan\theta_1 - X_2 \tan\theta_2 - R_1 + R_2}{\tan\theta_1 - \tan\theta_2} \quad (13a)$$

$$\tilde{R} = R_1 + \left\{ \frac{X_1 \tan\theta_1 - X_2 \tan\theta_2 - R_1 + R_2}{\tan\theta_1 - \tan\theta_2} - X_1 \right\} \tan\theta_1 \quad (13b)$$

The point midway between points 1 and 2, and the flow properties at this point, are then found from:

$$X_{12} = \frac{X_1 + X_2}{2}, \quad R_{12} = \frac{R_1 + R_2}{2} \quad (14a, b)$$

$$\rho_{12} = \frac{\rho_1 + \rho_2}{2}, \quad V_{12} = \frac{V_1 + V_2}{2} \quad (14c, d)$$

The density variation is found along the streamline defined by the straight line passing between the intersection point (\tilde{X}, \tilde{R}) and the point midway between 1 and 2 (X_{12}, R_{12}) . The cross-sectional area of the streamtube formed by the streamlines passing through points 1 and 2 varies as $R(R - \tilde{R})$, where R is the radial coordinate at an arbitrary point along the streamline. The density at this arbitrary point is then:

$$\rho = \rho_{12} \frac{R_{12} (R_{12} - \tilde{R})}{R(R - \tilde{R})} \quad (15)$$

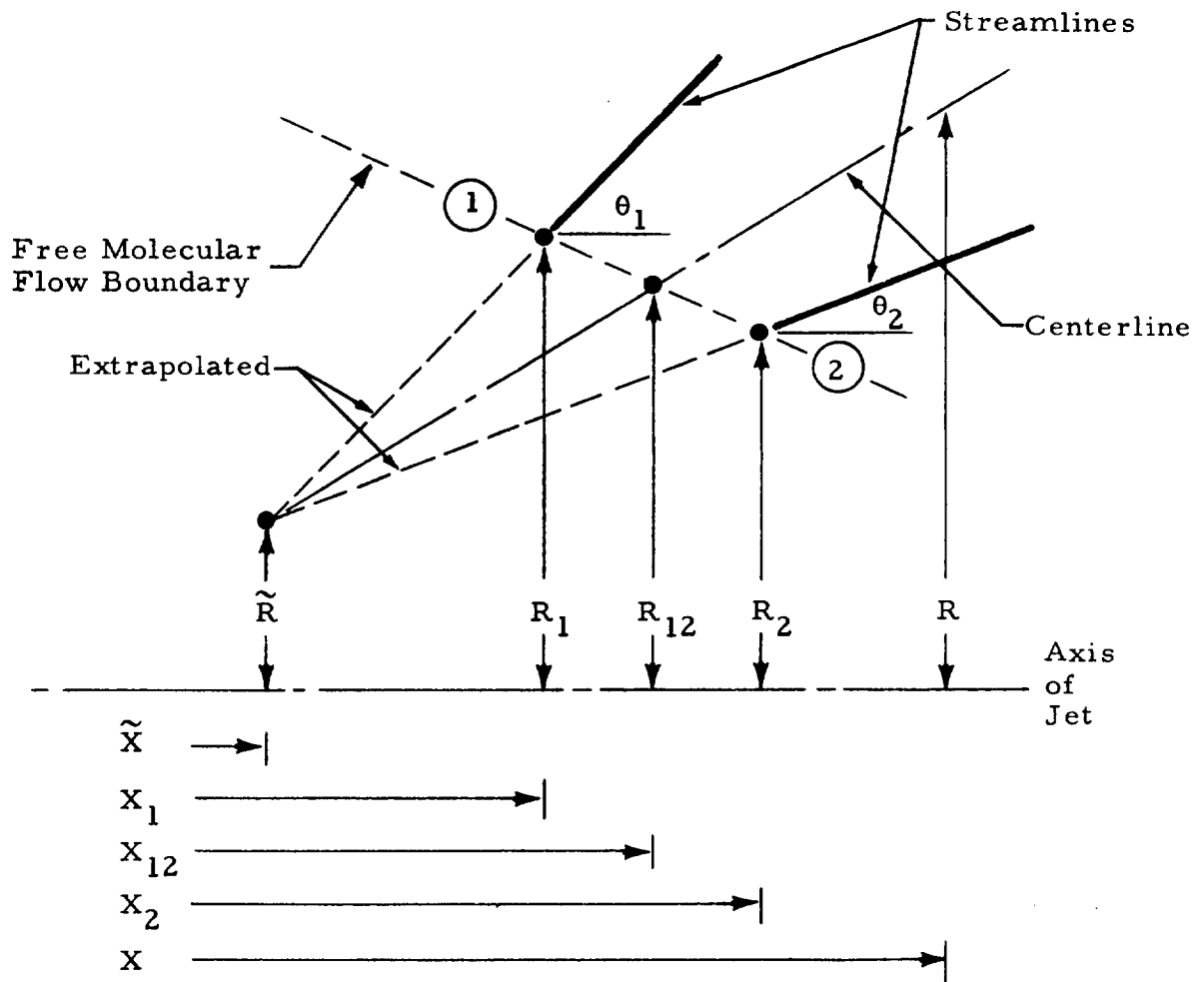


Figure 1 - Streamline Divergence in Free Molecular Flow Field

where

$$R = R_{12} + \frac{(X - X_{12})}{(X_{12} - \tilde{X})} (R_{12} - \tilde{R}) \quad (16)$$

When the intersection point (\tilde{X}, \tilde{R}) is on the jet axis, the expansion is the same as a spherical source flow expansion, as was discussed in Section 2. The above equations, however, are valid for any arbitrary diversion of the free molecular streamlines and do not depend on an assumed approximation to spherical source flow.

Section 4

CONCLUSIONS

The method of approach described in this report is considered to be the one most likely to yield reasonably accurate results within a reasonable period of time. It is straightforward in concept and may be readily incorporated into existing method-of-characteristics computer programs. Lockheed recommends, however, that consideration be given to developing the more rigorous approach of Hamel and Willis (Reference 11) and Edwards and Cheng (Reference 12) for use in plume flowfield calculation techniques.

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